Lebanese University
Faculty of Information
Section 1 Unesco

الجامعة اللبنانية
كلية الإعلام
الفرع الأول

1st Semester
Instructor : Dr. Abbas Rammal
Duration : 90 minutes

Course of Mathematics
Calculus

Exercise I :

Let f be a function defined near 0 by :

$$f(x) = \frac{\ln(1+\operatorname{sh}x) + e^{x\operatorname{ch}x} - \sqrt{1+x^3}}{x}.$$

1. Give the finite expansion near 0 to order 3 of

$$\ln(1+\operatorname{sh}x), \quad e^{x\operatorname{ch}x} \quad \text{and} \quad \sqrt{1+x^3}.$$

2. Deduce the finite expansion near 0 to order 2 of f. $2 + \frac{2}{3}x^2 + x^2\varepsilon(x)$
3. Show that f can be extended by continuity at $x = 0$ and give its extension g.
4. Show that g is differentiable at 0 and determine $g'(0)$.
5. Determine the equation of the tangent (T) at the point of abscissa $x = 0$ to the curve (C) of g, and determine the relative position of (T) with respect to (C) in a neighborhood of the point of abscissa $x = 0$

Exercise II :

1. Let

$$J = \int_0^{\frac{\sqrt{2}}{2}} \frac{u^2}{1-u^2}\,du$$

(a) Find the real numbers a, b and c such that

$$\frac{u^2}{1-u^2} = \frac{a}{1-u} + \frac{b}{1+u} + c$$

(b) Deduce that

$$J = \frac{1}{2}\ln(3+2\sqrt{2}) - \frac{\sqrt{2}}{2}$$

2. Calculate

$$I = \int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{\cos x}\,dx$$

Setting $u = \sin x$

3. Calculate

$$K = \int_1^3 x^2 \ln(x)\,dx$$

$\frac{x^3}{3}\ln x - \frac{x^3}{9} + C$

Exercise III :

Let f be the function defined by :

$$f(x) = \frac{\ln(\cos x) + x\sin x - \sqrt{1+x} + \frac{x}{2} + 1}{x^2}$$

Calculate $\lim_{x\to 0} f(x)$.

Exercise III :

1. Let $x \geq 0$. Applying the Mean value theorem for the function $f(t) = \arctan(t)$ over $[x, x+1]$, show that :

$$\frac{1}{x^2+2x+2} < \arctan(x+1) - \arctan(x) < \frac{1}{1+x^2}.$$

2. Deduce $\lim_{x\to+\infty} x(\arctan(x+1) - \arctan(x))$. o