Lebanese University
Faculty of Information
Section 1 Unesco

الجامعة اللبنانية
كلية الإعلام
الفرع الأول

الجامعة اللبنانية
UNIVERSITE LIBANAISE

1st Semester
Instructor : Dr. Abbas Rammal
Duration : 90 minutes

Course of Mathematics
Calculus

Exercise 1 . (10 points) Find the derivative of each of the following functions :

1. $f(x) = x\ln(e^x + 1)$.
2. $g(x) = \arctan(\frac{x+1}{x})$.

Exercise 2 . (15 points) Let f be the real function defined by :

$$f(x) = \begin{cases} x + a + \sqrt{x^2 + x + 1} & if \quad x < -1 \\ ax - b + a & if \quad -1 \leqslant x \leqslant 1 \\ bx^2 - 2x & if \quad x > 1 \end{cases}$$

where a and $b \in \mathbb{R}$.
Determine a and b so that f be continuous at $x = -1$ and $x = 1$.

Exercise 3 . (15 points) Let f be the function defined on $I =]0, +\infty[$ by $f(t) = e^{\frac{1}{t}}$.

1. Show that $f'(t) = \frac{-1}{t^2}e^{\frac{1}{t}}, \forall\, t \in I$.
2. Let $x > 0$. By applying the Mean Value Theorem to the function $f(t)$ on $[x, x+1]$, show that

$$\frac{1}{(x+1)^2}e^{\frac{1}{x+1}} < e^{\frac{1}{x}} - e^{\frac{1}{x+1}} < \frac{1}{x^2}e^{\frac{1}{x}}.$$

Exercise 4. (20 points)

1. Calculate

$$I = \int \frac{x+4}{x^2 + 2x + 5}\,dx$$

2. Determine the real constants A, B and C such that

$$\frac{1}{(1+t^2)(1+t)} = \frac{A}{1+t} + \frac{Bt + C}{1+t^2}$$

3. Calculate

$$J = \int \frac{1}{(1+t^2)(1+t)}\,dt$$

$$K = \int \frac{\sin x}{(1+\cos^2 x)(1+\cos x)}\,dt$$
