

Partial Exam

1st Semester

Instructor : Dr. Abbas Rammal

Duration : 60 minutes

Course of Mathematics

Calculus

Exercise I :

Given $a \in \mathbb{R}$. Consider the function f , defined by :

$$f(x) = \begin{cases} \sqrt{1+x}, & \text{if } x > 0; \\ \frac{x+a}{x-2}, & \text{if } x < 0. \end{cases}$$

1. Determine a such that f is extendable by continuity to $x = 0$. Let g be the extension function of f .
2. Determine $g'(x)$ for $x \neq 0$.
3. Is the function g differentiable at $x = 0$? Justify your answer.

Exercise II :

Let f be a function defined by :

$$f(t) = \ln \frac{1-t}{1+t} \quad \text{where } t \in [0, 1]:$$

1. Let $x \in]0, 1[$. Applying the Mean Value Theorem to f over $]0, x[$, show that exists $c \in]0, x[$ such that

$$f(x) = -\frac{2x}{1-c^2}$$

2. Deduce that $(1-x) \exp(2x) < 1+x$ for $x \in]0, 1[$.