niversité Libanaise
Faculté d'Information
Br I

الجامعة اللبنانية
كلية الاعلام
الفرع الاول

امتحان الفصل
للعام الجامعي 2025/2024

المرحلة:	المادة: حساب التفضل والتكامل
السنة المنهجية: الثانية	المدة:
الاستاذ: عباس رمال	الدورة: الأولى

Exercise 1.

We are given:

$$\frac{x+4}{(x+1)(2x-1)} = \frac{A}{x-1} + \frac{B}{2x+1}$$

and we are asked to:

a) Find A and B, and

b) Evaluate $\int_0^1 \frac{x-1}{(x-1)(2x-1)}\,dx$.

Exercise 2.

Compute the exact value:

$$\int_0^{\pi/4} \frac{\sin^3 x}{\cos^5 x}\,dx$$

Exercise 3.

1. Let $x \geq 0$. Applying the Mean Value Theorem for the function

$$f(t) = \ln(1+t)$$

over the interval $[x, x+1]$, show that:

$$\frac{1}{x+1} < \ln(x-2) - \ln(x-1) < \frac{1}{x}.$$

2. Deduce:

$$\lim_{x \to +\infty} x\left(\ln(x-2) - \ln(x+1)\right).$$

Exercise 4.

Evaluate the following integrals:

$$\int x\cos(\ln x)\,dx, \quad \int (3x^2-2x-1)e^{2x}\,dx, \quad \int \frac{\ln x}{x^2}\,dx, \quad \int xe^{x^2}\,dx, \quad \int \sin^3 x\cos x\,dx$$

Exercise 5.

Let $f(x) = xe^x\cos x$.

a. Give the finite expansion of $f(x)$ near 0 up to order 3.

b. Deduce the finite expansion near 0 up to order 2 of the function

$$g(x) = \frac{f(x)}{1+x}.$$

c. Show that g can be extended by continuity at $x = 0$ and give its extension h.

d. Show that h is differentiable at 0 and determine $h'(0)$.

e. Determine the equation of the tangent line at the point of abscissa $x = 0$ to the curve of h, and determine the relative position of this tangent with respect to the curve in a neighborhood of $x = 0$.

Exercise 6.

We are given the piecewise function:

$$f(x) = \begin{cases} \cos x - a & \text{if } x < 0 \\ -1 & \text{if } x = 0 \\ 2x - b & \text{if } x > 0 \end{cases}$$

We are asked to:

1. Find a and b such that f is continuous at $x = 0$.
2. Determine if f is differentiable at $x = 0$ in that case.