Lebanese University
Faculty of Information
Section 1 Unesco

الجامعة اللبنانية
كلية الإعلام
الفرع الأول

1st Semester
Instructor : Dr. Abbas Rammal
Duration : 90 minutes

Final Exam 2022-2023
Course of Mathematics
Calculus

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**Exercise I :**

Let $f$ be a function defined on $\mathbb{R}$ by :

$$f(x) = \begin{cases} \sqrt{x^2 - x + 1} & \text{if } x \leq 0; \\ \frac{1}{x+1} & \text{if } x \geq 0. \end{cases}$$

1. Show that $f$ is differentiable at $x = 0$.
2. Calculate $f'(x)$ for every $x \neq 0$.
3. Show that there exists a real number $c \in ]-2, 0[$ such that

$$f(-2) - f()0 = -2f'(c)$$

Determine $c$.

**Exercise II :**

1. Let

$$J = \int_0^{\frac{\sqrt{2}}{2}} \frac{u^2}{1-u^2}\, du$$

(a) Find the real numbers $a$, $b$ and $c$ such that

$$\frac{u^2}{1-u^2} = \frac{a}{1-u} + \frac{b}{1+u} + c$$

(b) Deduce that

$$J = \frac{1}{2}\ln(3 + 2\sqrt{2}) - \frac{\sqrt{2}}{2}$$

2. Calculate

$$I = \int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{\cos x}\, dx$$

Setting $u = \sin x$

3. Calculate

$$K = \int_1^3 x^2 \ln(x)\, dx$$

Turn the page...

**Exercise III :**

1. Find the finite expansions of order **3**, in the neighborhood of $x = 0$, of the following function :

$$f(x) = \frac{\ln(\cos x)}{\cos x - 1}$$

2. Let $f$ be a function defined by

$$f(x) = \frac{e^{-x} \sin x}{x}$$

(a) Write down the finite expansion of $f$ to order **2** in the neighborhood of $x = 0$. Show that $f$ is extendable by continuity to $x = 0$, with function $g$ as its extension.

(b) Determine the tangent to the curve of $g$ at the point of abscissa **0**, as well as its position relative to that curve near that point.

Good work...