Lebanese University
Faculty of Information
Section 1 Unesco

الجامعة اللبنانية
كلية الإعلام
الفرع الأول

8769

1st Semester
Instructor : Dr. Abbas Rammal
Duration : 90 minutes

Course of Mathematics
Calculus

Exercise 1 : ► 18 points

Let f be a function defined near 0 by :

$$f(x) = \frac{\ln(1+\text{sh}x) + e^{x\text{ch}x} - \sqrt{1+x^3}}{x}.$$

1. Give the finite expansion near 0 to order 3 of

$$\ln(1+\text{sh}x), \quad e^{x\text{ch}x} \quad \text{and} \quad \sqrt{1+x^3}.$$

2. Deduce the finite expansion near 0 to order 2 of f.
3. Show that f can be extended by continuity at $x = 0$ and give its extension g.
4. Show that g is differentiable at 0 and determine $g'(0)$.
5. Determine the equation of the tangent (T) at the point of abscissa $x = 0$ to the curve (C) of g, and determine the relative position of (T) with respect to (C) in a neighborhood of the point of abscissa $x = 0$

Exercise 2 : ► 12 points

1. Calculate

$$I = \int \frac{x+4}{x^2+2x+5}\,dx$$

2. Determine the real constants A, B and C such that

$$\frac{1}{(1+t^2)(1+t)} = \frac{A}{1+t} + \frac{Bt+C}{1+t^2}$$

3. Calculate

$$J = \int \frac{1}{(1+t^2)(1+t)}\,dt$$

$$K = \int \frac{\sin x}{(1+\cos^2 x)(1+\cos x)}\,dt$$

Exercise 3 : ► 15 points

1. Let $x \geq 0$. Applying the Mean value theorem for the function $f(t) = \arctan(t)$ over $[x, x+1]$, show that :

$$\frac{1}{x^2+2x+2} < \arctan(x+1) - \arctan(x) < \frac{1}{1+x^2}.$$

2. Deduce $\lim\limits_{x\to+\infty} x(\arctan(x+1) - \arctan(x))$.

Exercise 4 : ► 15 points

Let f and g be the functions defined on $[1, \infty[$ by

$$f(x) = \arcsin(\frac{1}{x}) \text{ and } \quad g(x) = \arccos(\frac{1}{x}).$$

1. Calculate $f'(x)$ $\forall x \in]1, +\infty[$.
2. Calculate $g'(x)$ $\forall x \in]1, +\infty[$.
3. Show that $f(x) + g(x) = k$ $\forall x \in [1, +\infty[$, where k is a constant to be determined.

Good luck