1st Semester
Instructor : Dr. Abbas Rammal
Duration : 90 minutes

Final Exam 2020-2021
Course of Mathematics
Calculus

Linear Calculus

Exercise I :

Consider the function f, defined by :

$$f(x) = \begin{cases} \sqrt{1+x} & \text{if } x \geq 0; \\ \sqrt{1-x} - x, & \text{if } x < 0. \end{cases}$$

1. Show that f is continuous at $x = 0$.
2. Study the differentiability of f at $x = 0$.
3. Cloud the Mean Value Theorem be applied to f over $[-1, 1]$?

Consider the differential form :

$$F = (3xy - y^2)dx + x(x - y)dy = 0$$

① Is the differential form F is exact ?
② Determine an integrating factor $\mu(x)$ such that μF is exact (to be verified).
③ Find the general solution of the differential form $\mu F = 0$.

Exercise II :

1. Let

$$J = \int_0^{\sqrt{3}} \frac{u^2}{1-u^2}\, du$$

(a) Find the real numbers a, b and c such that

$$\frac{u^2}{1-u^2} = \frac{a}{1-u} + \frac{b}{1+u} + c$$

(b) Deduce that

$$J = \frac{1}{2}\ln(3 + 2\sqrt{2}) - \frac{\sqrt{2}}{2}$$

2. Calculate

$$I = \int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{\cos x}\, dx$$

Setting $u = \sin x$

3. Calculate

$$K = \int_1^3 x^2 \ln(x)\, dx$$

Turn the page...

Exercise III :

1. Find the finite expansions of order 3, in the neighborhood of $x = 0$, of the following function :

$$f(x) = \frac{\ln(\cos x)}{\cos x - 1}$$

2. Let f be a function defined by

$$f(x) = \frac{e^{-x} \sin x}{x}$$

(a) Write down the finite expansion of f to order 2 in the neighborhood of $x = 0$. Show that f extendable by continuity to$x = 0$, with function g as its extension.

(b) Determine the tangent to the curve of g at the point of abscissa 0, as well as its position relativ to that curve near that point.

Good work..